H = thermal effectiveness factor = $\frac{C_g\rho_g(T_0 - T_s)}{\rho_s\lambda m_0}$
h' = volumetric heat transfer coefficient
L = bed depth
M = dimensionless local moisture content = m/m_0
$\overline{M}$ = dimensionless average moisture content
$\overline{M}_F$ = dimensionless final average moisture content
$M^0{}_F$ = dimensionless minimum permissible moisture content
$M^L{}_F$ = dimensionless maximum permissible moisture content
m_0 = initial moisture content
n = exponent
P = power consumption
s_m = state variable
T_0 = inlet gas temperature
T_s = adiabatic saturation temperature
t = time
t_F = time for passage of a unit mass of dry solid
U = superficial gas velocity
Z = dimensionless distance = z/L

Greek Letters

β = volumetric Stanton number = $h'L/U\,\rho_g C_g$
ϵ = bed void fraction
ϕ = dimensionless group = $\frac{U}{2\epsilon E}\left[1 - \left(1 + \frac{4h'\epsilon E}{C_g\rho_g U^2}\right)^{1/2}\right]$
Φ = dimensionless time = Ut/L
λ = latent heat of vaporization
ρ_g = gas-phase density
ρ_s = bulk density of bed
μ_g = gas-phase viscosity
θ = dimensionless temperature = $(T - T_s)/(T_0 - T_s)$

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Feedforward Computer Control of a Class of Distributed Parameter Processes

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A finite difference technique has been developed which is suitable for the derivation of feedforward control algorithms for the control of distributed parameter processes. The method embodies both steady state and transient feedforward compensation.

An experimental study, involving closed loop, on line, analogue computer control of a simple distributed parameter process proves the feasibility of the finite difference technique. The process under study is a steam water heat exchanger subject to inlet temperature forcing and controlled by flow rate manipulation. The new technique yielded significantly improved results when compared with the performance of a conventional two-mode feedback controller, or a linearized feedforward control algorithm.

The use of analogue and particularly digital computers within the past 15 yr. has projected theoretical developments in the process control field far ahead of the experimental. Simulation of equations has, to a large extent, displaced the laboratory verification of advanced control schemes.

The need for experimental evidence in this area is supported by the following arguments: mathematical models cannot exactly predict real process behavior, and the extent of the difference cannot be determined except by experiment; process noise can preclude or seriously limit the effectiveness of sophisticated control schemes such as Liapunov (*1*, *2*) or model reference adaptive control (*3*, *4*) which can require the differentiation of signals; real transducers, control elements (valves, etc.), will be of limited accuracy and may be subjected to nonlinearities such as sticking and hysteresis (*5*) which are not taken

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into account in the analysis of control system performance. Although the above list is not complete, it points up the need for experimental verification of some of the advanced process control schemes in the literature.

The purpose of this paper is to present a new scheme for feed-forward computer control of a class of distributed parameter processes. In addition, quantitative results, obtained from several on-line computer control studies, are presented to show the improvement in system performance when this scheme is utilized. The specific process under consideration is a steam to water concentric pipe heat exchanger followed by a measurement dead time.

Although considerable theoretical work on feedforward control has appeared in the literature, relatively little experimental work has been done. Bollinger and Lamb (*8*) synthesized a theory of feedforward control for linear systems. The behavior of chemical reactors (*9, 10, 12*) and distillation columns (*31*) with linearized process characteristics has also been studied.

Nonlinear feedforward control in the steady state gain portion of the feedforward controller was considered by Luyben (*11*) and MacMullen and Shinskey (*13*) for distillation columns. Luyben (*14*) presented nonlinear feedforward control algorithms for lumped parameter chemical reactors and discussed some of the practical and mathematical problems which can arise when feedforward control is applied to distributed parameter systems. Buckley (*6*) also points out the difficulty of controlling distributed parameter systems by conventional feedback methods. This difficulty is compounded when the process is nonlinear. Because feedforward control can be used for the control of nonlinear processes, and since it probably offers the greatest potential for solving the dead time problem, a general approach to the feedforward control of distributed parameter systems is an important area to be investigated.

FEEDFORWARD CONTROL

Feedforward control can conveniently be divided into two separate categories: steady state and dynamic. The distinction lies in whether the feedforward control algorithm is synthesized from the steady state or transient operating equations of the process under consideration. This paper will be concerned with both dynamic and steady state feedforward control of distributed parameter systems.

The concept of feedforward control implies the use of a process model. Disturbances entering the process, for which feedforward compensation is desired, are transmitted to the model. Control action is calculated in the model, and the information is transmitted back to the process in order to meet the control objective.

For lumped parameter processes, the feedforward control algorithms can be synthesized by setting the process outlet dependent variables equal to the desired functions and by solving for the manipulated variables from the process operating equations.

Three methods may be postulated for synthesizing transient feedforward control algorithms for distributed parameter systems. These are: obtaining an exact solution to the general process equations, linearizing the process equations and obtaining an exact solution to the linearized equations, and employing a finite difference technique and a computer to solve the equations.

The following example will serve to illustrate these methods. The specific process under consideration is a concentric pipe, steam to water, heat exchanger. The dynamics of this process have been studied with recourse to various linearization techniques by several investigators (*16, 23, 24, 29*). Koppel (*15, 17*) has shown that the isothermal plug flow reactor is also described by the same equations as the heat exchanger. The partial differential equations governing the process and its boundary conditions are (*15*):

$$\frac{\partial \phi}{\partial t} + (1 + r)\frac{\partial \phi}{\partial x} = (1 + r)^b - (1 + r) \tag{1}$$

$$\phi(x, 0) = 0 \tag{2}$$

$$\phi(0, t) = \phi_1(t) \tag{3}$$

In these equations, $r(t)$ represents a normalized velocity deviation from steady state, while $\phi(x, t)$ is the transformed, normalized, process dependent variable. The exponent b is related to the flow rate dependence of the overall coefficient when the equations are used to describe the heat exchanger. For the chemical reactor, b is zero, and, for a heat exchanger, $0 \leq b \leq 0.8$.

In this study, the control objective is to maintain the outlet temperature of a heat exchanger at a steady state operating point, despite variations occurring in the inlet temperature, by manipulation of the flow rate through the exchanger. None of the feedforward techniques which are discussed in this work is limited to a steady state criterion. A time varying criterion could easily be incorporated into any of the methods. In terms of the normalized variables, the control objective becomes

$$\phi(1, t) = 0 \tag{4}$$

The first method postulated for synthesizing the feedforward control algorithm is to seek an analytical solution to Equation (1). Koppel (*15*) has presented such a solution for the case where the term $(1 + r)^b$ is replaced by its linear Taylor series approximation. Ray (*18*) presented a very general solution for the case of a heat exchanger, without recourse to any linearization. Neither solution, however, lends itself readily to the feedforward control problem discussed above. The reason for this is that there is great difficulty in obtaining an explicit relationship for the flow rate in terms of the inlet temperature disturbance. In addition, analytical solutions to nonlinear or parametrically forced linear partial differential equations are quite rare, so that this particular method would be of limited utility in the general case.

The second method involves the linearization of the process equations followed by an analytical solution of the linearized equations. Koppel (*15*) has shown that, for small variations in r and $\partial\phi/\partial x$, the product term $r(\partial\phi)/(\partial x)$ in Equation (1) may be ignored. Furthermore, if the term $(1 + r)^b$ is replaced by its linear Taylor series approximation $(1 + br)$ and substituted into Equation (1), one obtains

$$\frac{\partial \phi}{\partial t} + \frac{\partial \phi}{\partial x} = (b - 1)r \tag{5}$$

Employing LaPlace transforms and applying the control objective, one can write

$$r(s) = \left[r(s) + \frac{s}{1 - b}\,\phi_1(s) \right] e^{-s} \tag{6}$$

for the feedforward control scheme.

Thus, the present value of the control variable should equal the value of the control variable which existed one dead time in the past plus $(1/1 - b)$ times the derivative of the disturbance which existed one dead time in the past. In this way, Equation (6) takes into account the memory of the specific distributed parameter process considered.

The third method, and the one this paper is primarily concerned with, involves discretizing the spatial dimensions of the partial differential equations governing the process. The result of this finite differencing method, in the general case, is a set of nonlinear ordinary differential equations in time. Using a forward finite difference (*25*) on Equation (1) and solving for the derivative with respect to time, one obtains

$$\frac{d\phi_{n+1}}{dt} = -(1+r)\frac{\phi_{n+1}-\phi_n}{\Delta x} + (1+r)^b - (1+r) \tag{7}$$

The process is now modeled as a series of stirred tanks whose mathematics are described by a set of ordinary differential equations with variable coefficients. In Equation (7), n varies between 0 and $m-1$, so that ϕ_m refers to the temperature or concentration related dependent variable leaving the last stirred tank. Applying the control objective, $\left[\ \phi_m = \phi(1,t) = 0, \text{ from which } \frac{d\phi_m}{dt} = 0\ \right]$, and solving for the flow rate, one obtains from the final stirred tank

$$r(t) = \left(\frac{\Delta x - \phi_{m-1}}{\Delta x}\right)^{\frac{1}{b-1}} - 1 \tag{8}$$

With an analogue computer, the quantity ϕ_i is sensed continuously, and the values of $\phi_2, \phi_3, \ldots \phi_{m-1}$ are generated in parallel [by using Equation (7)]. At the same time, the computed value of ϕ_{m-1} is used in Equation (8) to generate the value of the manipulative variable $r(t)$ which is used in Equation (7).

Thus, a technique which has found a great deal of success in the solution of partial differential equations by using analogue computers (*19, 20, 21*) can be employed for deriving feedforward control algorithms for computer control of distributed parameter processes. Although an analogue computer was employed in this work, the particular form of Equations (7) and (8) suggest the use of a hybrid control computer. Thus, all nonlinear function generation and algebraic manipulations could be performed in the digital portion of the hybrid computer, while integration of the differential equations could be programmed in the analogue section.

The analogy between this method and what is done in deriving the feedforward algorithm in lumped parameter systems can be explained with reference to Figure 1.

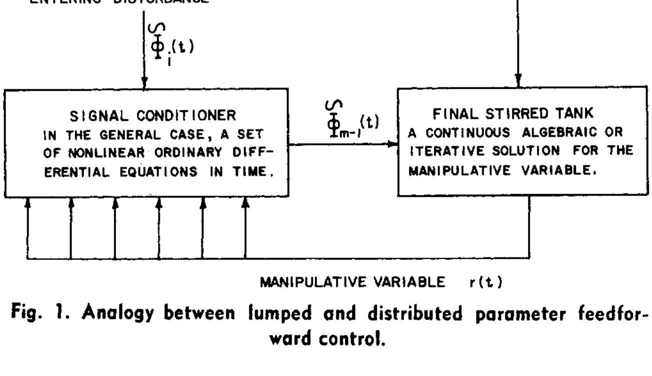

Fig. 1. Analogy between lumped and distributed parameter feedforward control.

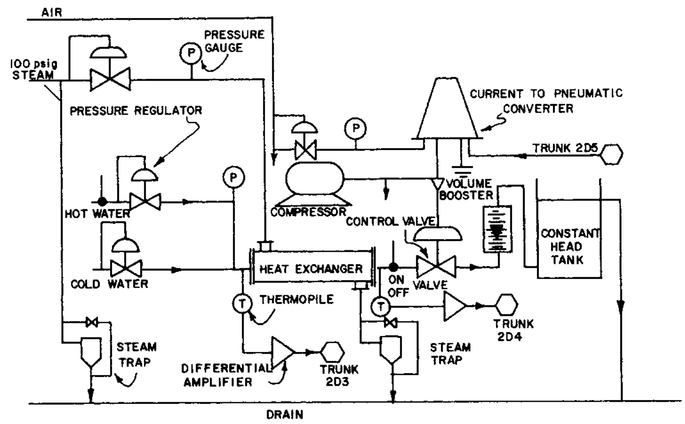

Fig. 2. Flow diagram of heat exchanger system.

The additional element required is the block marked signal conditioner. In the signal conditioner, the disturbance is modified in a manner approximating what occurs in the distributed system being controlled. The output of the signal conditioner is then fed to the last stirred tank in the series. Here the analogy between the derivation of the feedforward control algorithm for lumped vs. distributed systems becomes obvious. The output of the signal conditioner is equivalent to the disturbance entering the lumped parameter system. Although only a single dependent variable has been used in the above example, extension into the area of multivariable feedforward control can easily be done. Here, however, one may be faced with simultaneously solving a set of nonlinear algebraic equations.

The finite difference technique, as presented, will work for cases where the response of the process to the manipulative variable is very fast, such as the response of a heat exchanger to flow rate changes. For cases where there is a dead time associated with the action of the manipulative variable, the following modification of the technique can be proposed. The finite difference equations can be solved in high speed (approximately one hundred times real time) repetitive operation for time ranging between the present t and the present plus the dead time of the manipulative variable $t + \theta$. The present value of the manipulative variable can then be chosen such that the control criterion is satisfied at $t + \theta$. This calculation of the manipulative variable would, by its nature, be an iterative one. However, the high operating speed of an analogue or hybrid computer should permit one to instantaneously solve for the present value of the manipulative variable. This proposed scheme would fail for those cases where disturbances could enter the process and effect its output in a time duration which was less than the dead time of the manipulative variable. However, in such cases feedforward control would not work anyway, and a new manipulative variable should be chosen.

Finally, because of the assumptions made in the derivation of the model, it is apparent that differences will exist between the model and the process so that a certain amount of feedback control action will be necessary in the implementation of the feedforward controller.

EXPERIMENTAL INVESTIGATION

A flow diagram of the equipment used is shown in Figure 2. The inner and outer pipes of the concentric pipe heat exchanger were constructed of 3/4 and 1 1/2 in. nominal diameter, type L copper water tubing. The exchanger had an effective heated length of 14.25 ft. and a heated area of 2.92 sq.ft.

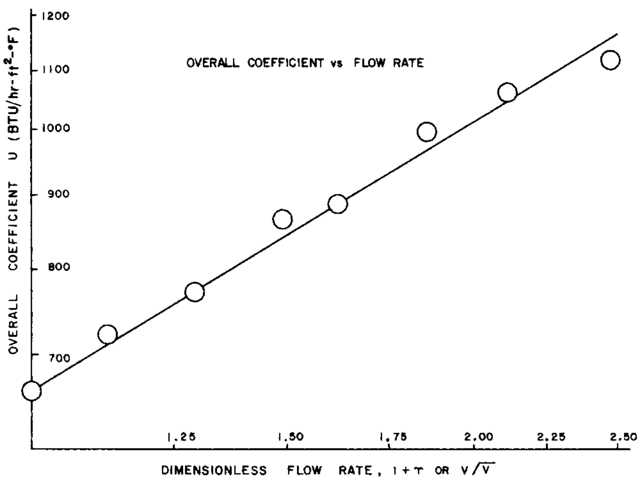

Fig. 3. Overall coefficient vs. flow rate.

Cold water flowed into the inner tube of the exchanger, while steam at 12.5 lb./sq.in. gauge condensed in the shell. An on-off valve at the inlet of the exchanger allowed hot water to be added to the incoming cold water stream in a mixing tee.

Two, three-element thermopiles were used to detect the inlet and outlet water temperature. In verifying the system dynamics, the probes were inserted at the center-line inlet and outlet points of the heat exchanger. For the control studies, the outlet side thermopile was relocated some distance downstream of the exchanger outlet. The reasons for relocating the probe are that the feedforward action can be observed relatively independent of the feedback controller; in practice, measurements from a sensor located at the discharge side of a process will not be available instantaneously; that is, there will be some measurement dead time. This is particularly true if the measured variable is composition. The steady state process and measurement dead times were about 4 and 20 sec., respectively. Approximate time constants for the thermopiles were determined by plunging them into boiling water and by recording the amplified electromotive force change on a storage oscilloscope. The time constants for both thermopiles were approximately 0.1 sec.

The inlet and outlet thermopile electromotive force was amplified in two differential amplifiers having gains of 1,000. The outputs of these amplifiers were wired into the appropriate trunk lines of an Applied Dynamics, AD-80 analogue computer. Within the temperature ranges involved, a linear, graphically determined inlet and outlet temperature-electromotive force relationship was developed. Inlet temperatures in the range 70° to 110°F. and outlet temperatures between 145° to 185°F. were observed. Data on the effect of fluid velocity on the overall coefficient of heat transfer were taken in order to determine the constants in the relationship

$$U = \overline{U}\,(1 + r)^b \tag{9}$$

presented by Koppel (*15*) for flow forced heat exchangers. With the process on line, inlet and outlet temperatures were integrated in the computer over a measured time interval and true time averaged temperatures determined for several different flow rates. The constant b was determined graphically to be 0.57, while $\overline{v}$ was chosen to be 3.55 ft./sec., for which $\overline{U}$ = 688 B.t.u./(hr.) (sq.ft.) (°F.). A plot of these data is shown in Figure 3.

In the control studies, the computer calculated flow rate voltage was transduced to a proportional current signal by using conventional analogue equipment, two precision (±1%) 500 ohm resistors and a 0.0047 μf capacitor. The complete circuit for the analogue computer current source may be found elsewhere (*30*).

The current signal generated in the computer was transduced to a proportional pressure signal in a current to pneumatic converter. The output of the converter, in the 3 to 15 lb./sq.in. gauge range, was fed directly into a pneumatic volume booster supplied with filtered air at 160 lb./sq.in. gauge from a compressor. The volume booster output signal was used to drive a ¾ in. equipercentage pneumatic control valve located downstream of the exchanger. The function of the volume booster was to increase the valve inlet air velocity limit. Tests indicated that the volume booster was capable of decreasing the valve time constant from 6.5 to 0.5 sec. The position of the valve stem was measured continuously by means of a 10,000 ohm wire-wound potentiometer mounted on the valve body. For further details relating to the equipment used in this study, the reader is referred to (*26*).

On line experimentation was used to determine the actual Ziegler-Nichols settings for the process. In this way, the ultimate period was found to be P_u = 120 sec., while the ultimate gain was K_u = 2.47. For the system under proportional plus integral feedback control, the reset rate was T_r = 100, while the proportional gain was K_c = 1.11, and the integral constant was K_I = 0.0111. The feedback controller employed in this work was

$$P_c = K_c + K_I \int \epsilon dt \tag{10}$$

where the error is defined as

$$\epsilon = T_{sp} - T_0 \tag{11}$$

Several experiments were made with the system under pure feedback, linearized feedforward and feedback, and finite difference feedforward and feedback. In addition, a series of experiments were conducted in order to verify the dynamics of the finite difference model employed in this work. A comparison of the dynamic response of the process and model for flow forcing is shown in Figure 4. For these tests, a step change in input voltage, corresponding to a doubling of the process flow rate, was applied to the pneumatic valve. The forcing function for the model, however, was the signal from the potentiometer mounted on the control valve, which was transduced to dimensionless flow rate within the computer. Thus the model and process were subjected to essentially the same disturbance at the same time.

The process and model response to inlet temperature forcing are compared in Figure 5. In these experiments, the actual temperature entering the exchanger was transduced to ϕ_i within the computer and also used as the forcing function for the model. At the same time, the actual outlet temperature was converted continuously to ϕ_0 and recorded. The inlet temperature was changed from 70° to 100°F. and back again in this test.

The model employed in the verification of the dynamics consisted of five stirred tanks. Because of the capability of the computer to simulate the process in real time, simultaneous recording of the model and process output dependent variable was possible.

Four separate closed loop studies were made. Typical inlet temperature disturbances for these four cases are given in Figure 6. The closed loop process response to forcing by increasing and decreasing temperature disturbances is given in Figures 7, 8, and 9.

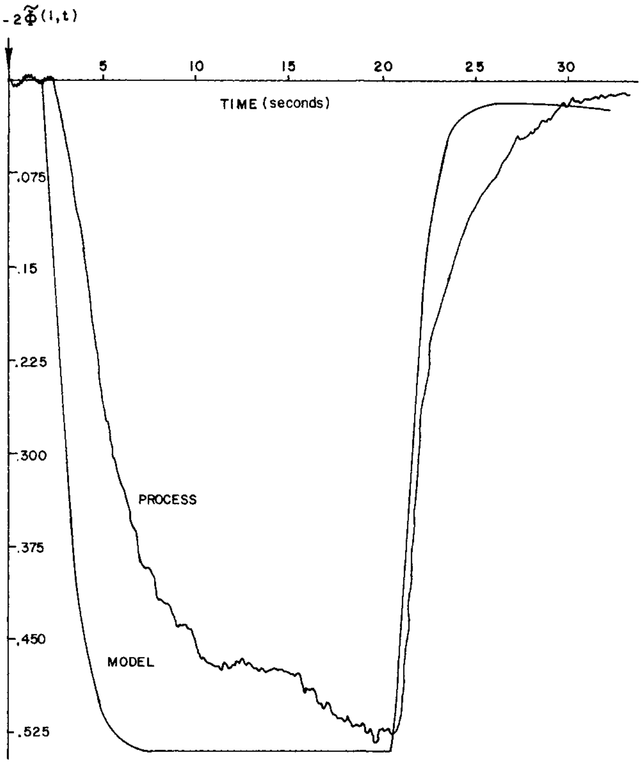

Fig. 4. Process vs. model response flow forcing.

COMPUTER CONSIDERATIONS

Because differentiation amplifies low level, high frequency noise present in all analogue computers, the following approximation to the transfer function of the derivative was employed:

$$\frac{Y(s)}{X(s)} = \frac{s}{1 + 0.1\,s} \tag{12}$$

Equation (12) represents a fairly good approximation to the derivative at frequencies below 1 rad./sec.

Methods for dead time simulation by using conventional analogue equipment are limited to the linear circuit approximation technique, many of which have a poor transient response. Process control applications, however, demand that approximations have both a good transient, as well as frequency response. The linear circuits presented recently by McAvoy (7) display the best combined transient and frequency response of those available and were used in the present work.

Although delay and differentiation are both linear operations, both are approximations, and the order of differentiation and delay influences the accuracy of the result obtained in programming Equation (6). It was found, in simulation studies, that differentiation followed by delay resulted in the greatest accuracy, at least for the particular approximations employed in this work.

OPEN LOOP VERIFICATION OF PROCESS DYNAMICS

The differences between the process and plant shown in Figures 4 and 5 are due primarily to the assumptions made in the derivation of the model. Except for the very minor lag introduced by the thermopiles, the dynamics of the control elements were not a factor contributing to the errors noted. The thermal lag accompanying flow or inlet temperature forcing in either direction is indicative of the importance of the metal wall capacitance, a factor neglected in this study.

Although approximately the same steady states were attained by the process and the model in flow forcing, there was an offset with temperature forcing. This offset is a consequence of the assumption that the overall coefficient is a function of flow rate only. Actually, the individual coefficient on the tube or water side is a relatively strong function of the bulk viscosity. Because of the large inlet temperature disturbance which was initiated, some offset was to be expected. The effect of the change in bulk temperature in closed loop control would not be as severe as the effect noted in Figure 5, because the exit temperature would be constrained to the region near its set point value. Rough calculations indicate a change in bulk temperature of 12% for the closed loop and 21% for the open loop, for inlet temperatures ranging between 70° and 100°F.

If the model were made more rigorous, by including the effects of wall capacitance, a more complex feedforward algorithm would result, and closer agreement between the process and model would be expected. However, this is not a specific limitation of the finite difference technique but rather a general limitation common to any of the methods described for the derivation of feedforward controllers. It should be noted that the complexity of the finite difference technique is not increased in this case, merely the size of the resultant algorithm and of course the hardware necessary for its implementation.

DISCUSSION OF RESULTS

Four different closed loop control schemes were investigated in this study. The process response to increasing and decreasing temperature forcing for the case of proportional plus integral feedback control is shown in Figures 7

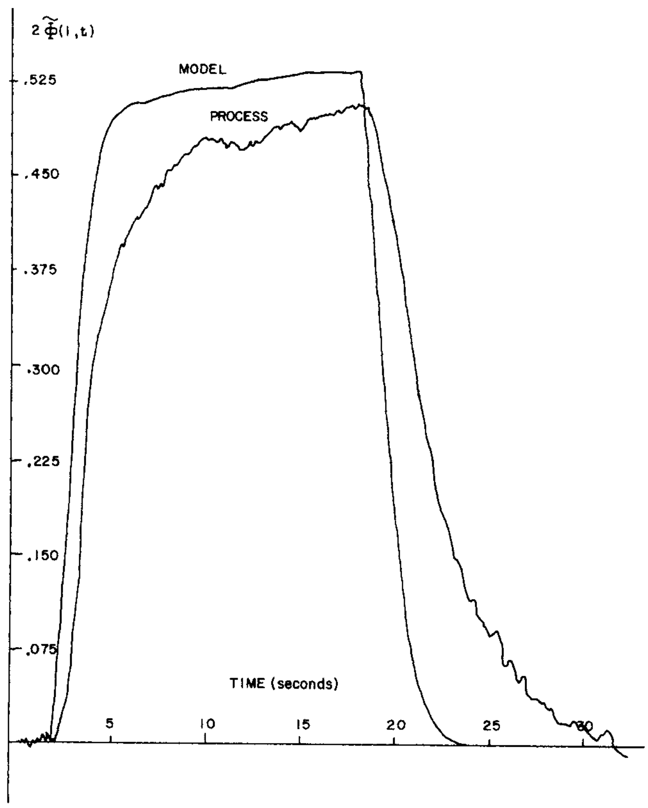

Fig. 5. Process vs. model response inlet temperature forcing.

and 8, respectively. It may be noted that considerable dissimilarity exists between the process responses shown, depending upon the direction of the forcing. Probably the most important factor contributing to the difference between the two responses is the variable dead time nature of the process. Because the Ziegler-Nichols settings represent average values determined over a limited range of flow rates, no one value of gain can satisfactorily compensate for disturbances of the type imposed in this work.

For the case of an increasing temperature disturbance, compensation is effected by increasing the process flow rate. Increasing the process flow rate decreases the dead time and thereby enhances the relative stability of the loop. However, the average Ziegler-Nichols settings become conservative at high flow rates, and the sluggish process response shown in Figure 7 results. Compensation for decreasing inlet temperatures results in the highly oscillatory response shown in Figure 8. In this case, the valve must close, thereby increasing the process dead time and decreasing the relative loop stability. In addition, the process is moving toward a more distributed system, which increases the feedback control difficulty.

With proportional and integral control, the maximum overshoot was approximately 15°F., while the settling time was about 100 sec. in the direction of increasing dead time and 240 sec. in the direction of decreasing dead time. Settling time is defined as the time required for the process output temperature to attain 2% of its final steady state value.

Also shown in Figures 7 and 8 are the results obtained when the finite difference method employing five stirred tanks was applied. Two subcases were studied: the finite difference feedforward method with Ziegler-Nichols, proportional plus integral feedback settings, and the finite difference method with the feedback settings relaxed to 60% of their original values. It may be noted that the effect of the variable dead time on the relative loop stability has been almost completely eliminated. Complete cancellation of the effect of the inlet temperature disturbance is, of course, not possible because of the inability of the model to describe the process exactly. Another factor preventing complete cancellation of the disturbance in any of the control studies is the fact that the dynamics of the valve and thermopiles were neglected.

With the finite difference feedforward control, a great improvement in system performance was noted. In fact, the feedback loop was found to contribute some undesirable oscillatory behavior to the total response. Relaxing the proportional gain of the feedback controller to the values previously specified resulted in the response curves labeled feedforward with tuned feedback and eliminated the oscillations. With feedforward control, the feedback loop gain can be decreased if a small deviation about steady state is allowable.

For a decreasing inlet temperature disturbance, the maximum overshoot observed with the finite difference method and the Ziegler-Nichols settings was about 6°F., or a 60% improvement over conventional feedback. In addition, the settling time was decreased to approximately 60 sec. For the case of an increasing inlet temperature disturbance, the maximum overshoot was reduced to 6°F., while the settling time was decreased to 20 sec. With the relaxed feedback settings, some trade off between overshoot, settling time, and decreased oscillatory behavior was observed, but the results are about the same as those described above.

The third case studied involved closed loop control by using a single stirred tank representation of the distributed system. Applying the finite difference technique in this case resulted in the following relationship for the flow rate dependence on the inlet transformed variable:

$$r(t) = (1 - \phi_i)^{\frac{1}{b-1}} \qquad (13)$$

Equation (13) represents a lumped, nonlinear feedforward algorithm for the heat exchanger. It should be noted that a different lumped algorithm results upon application of this technique to the partial differential equations of the process in temperature. If a step change in ϕ_i were applied, the flow rate predicted by the lumped model would also be a step change. Thus the lumped model does not take the dynamics of the process into account.

The results obtained when Equation (13) was programmed for on line control are shown in Figure 9. When subjected to increasing inlet temperature, the process begins to behave more like a lumped system, and fairly good control is realized. For decreasing inlet temperature disturbances, however, the lumped representation becomes less accurate because the distributed nature of the process becomes more pronounced. It should be noted that the valve responds slower in opening than in closing. This fact helps to account for the good performance of the lumped model to forcing with increasing inlet temperature, since the feedforward action is slowed up by the valve, and, in effect, the valve helps compensate for some of the process dynamics. The relatively fast response of the valve in closing, and the lack of dynamic compensation on the part of this control scheme, as the distributed nature of

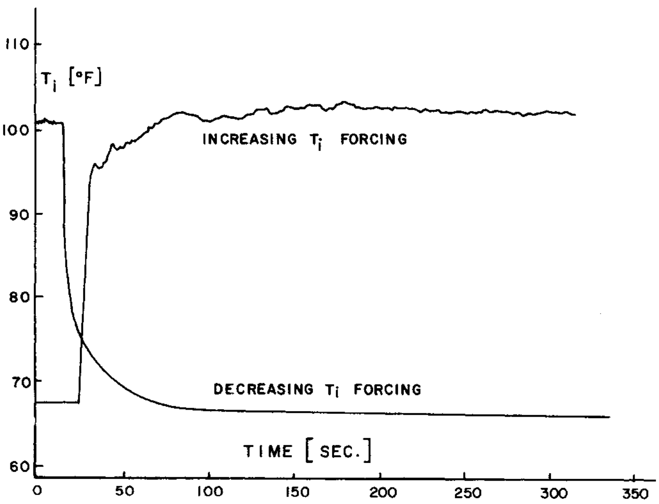

Fig. 6. Typical inlet temperature forcing functions.

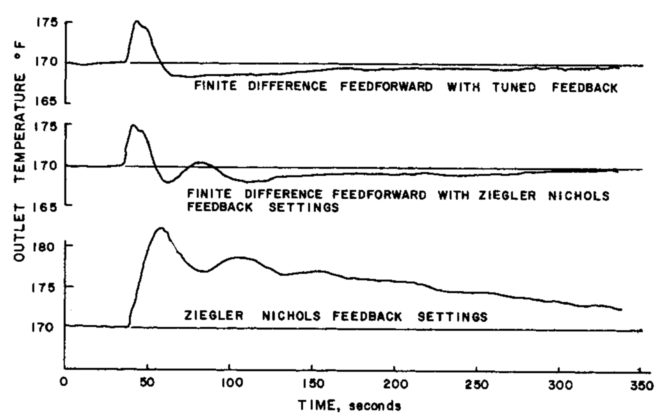

Fig. 7. System response comparison increasing inlet temperature forcing.

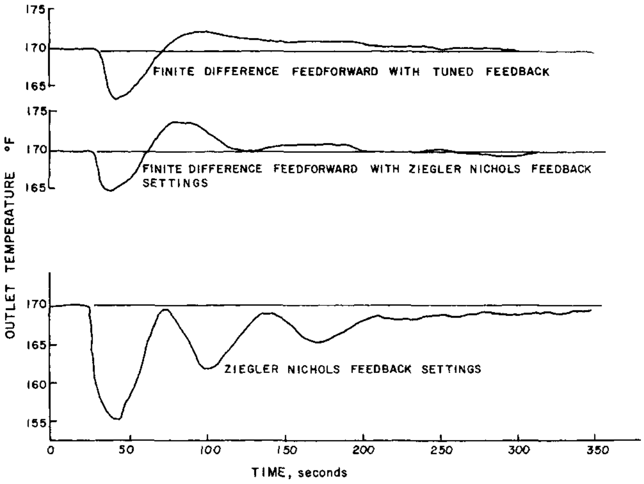

Fig. 8. System response comparison decreasing inlet temperature forcing.

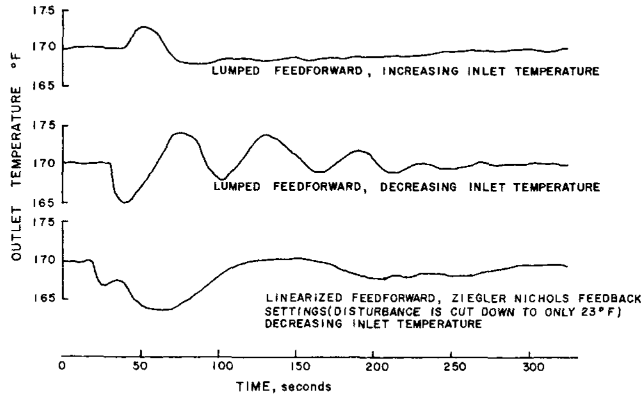

Fig. 9. Response of lumped and linearized feedforward controllers.

the process becomes more important, demonstrates that caution must be employed in the utilization of a single equation to describe a distributed parameter system.

The response of the linearized feedforward controller to decreasing inlet temperature is also shown in Figure 9. For these studies, the magnitude of the disturbance had to be decreased to 23°F. in order to obtain a stable response. Forcing with increasing inlet temperature caused the process to break into sustained oscillations, even for disturbances in the order of only 15°F. The poor performance of the linearized feedforward controller may be explained by a closer inspection of Equation (6). Rewriting Equation (6) in transfer function form, one obtains

$$\frac{r(s)}{\phi_i(s)} = \frac{1}{b-1}\,\frac{se^{-s}}{(1-e^{-s})} = \frac{s}{b-1}\,(e^{-s} + e^{-2s} + e^{-3s} + \ldots) \quad (14)$$

If, for the purpose of analysis, the initiated disturbance is considered to be a step change in the inlet transformed variable, then the flow rate predicted by Equation (14) would be a delayed pulse train. In effect, the process remembers the disturbance and continues to attempt corrective action even after the derivative of the disturbance has passed. Because this control was linearized about the low inlet temperature steady state condition, moving the process in that direction should provide the better response, as noted. Another inadequacy of this controller is its failure to predict a new steady state flow rate for a step change in ϕ_i. This is not the case of the algorithm determined by the finite difference method with one or five stirred tanks. The lumped representations will both give the correct new steady state flow rate for ϕ_i step changes, with the five stirred tank model introducing the additional benefit of transient control. It should be pointed out that the linearized model used to derive Equation (14) would not be expected to give good results when $r(t)\,\frac{\partial \phi}{\partial x}$ is large.

It is instructive to apply a one stirred tank lumped approximation to the linearized partial differential equation in ϕ. If this is done, one obtains

$$r = \frac{\phi_i}{1-b} \quad (15)$$

which is considerably different from the one stirred tank algorithm, Equation (13). Equation (15) does not predict the correct value of flow rate to compensate for changes in ϕ_i either. However, some compensation would be realized, and the process flow rate would move in the appropriate direction.

In order to show the problems which might result in the application of Equation (15) when large changes in the inlet variable occur, the steady state flow rates predicted by Equations (13) and (15) to step forcing in ϕ_i can be examined. For a step change of $\phi_i = 0.1$, Equation (13) predicts $r = 0.278$, while Equation (15) predicts $r = 0.232$, which is not too far in error. For the large disturbances imposed in this work, however, where $\phi_i = 0.356$, we find the exact flow rate to be $r = 1.88$, while the linearized model predicts $r = 0.828$. The foregoing serves to point up dramatically the danger inherent in the application of feedforward controllers derived from linearized process equations when the process is subjected to large disturbances.

CONCLUSIONS

An extension of feedforward control theory to include transient feedforward computer control of distributed parameter processes has been made. The new method is extremely flexible and permits the derivation of feedforward control algorithms for such processes by a finite difference technique. Two time scale computation is required when the process responds slowly or has a dead time associated with the action of the manipulative variable. The finite difference method can be used to control multivariable distributed parameter processes if care is taken in the choice of the manipulative variables. Time varying control objectives can easily be incorporated into the method, thus extending its utility to the cyclic control of distributed processes. The finite difference method appears to be generally applicable to the derivation of feedforward controllers for distributed systems, subject to the limitations given in the paper.

Experimental studies involving on line, analogue computer control of a distributed parameter steam to water heat exchanger confirm the theory. Of the methods studied, the finite difference method gave the best overall control performance. An advantage of the finite difference model is a relative insensitivity to process noise. In essence, the model acts to filter incoming noise much as the actual process does. It would be expected that the lifetime of final control elements would be increased if this method were employed. The finite difference method was successful in compensating for the time varying dead time of the process. The lumped parameter model showed distinct advantages over the linearized algorithm developed. Compared with the finite difference feedforward control scheme, the lumped representation could not compensate for the dynamics or the variable dead time as the distributed nature of the process became more important. Use of such a controller results in steady state and not transient control.

The linearized feedforward controller gave only fair results when the process was forced in the direction of the steady state operating point. When forced in the other direction, this controller caused the process to break into sustained oscillations. The use of controllers derived from linearized process equations should be viewed critically when the process is subjected to large disturbances.

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NOTATION

A_c = cross-sectional area available for flow, sq.ft.
A_H = heat transfer area based on outside tube area, sq.ft.
b = dimensionless empirical exponent ($b = 0.57$)
C_p = specific heat of water, B.t.u./(lb.)(°F.)
D_0 = outside tube diameter, ft.
K_c = feedback proportional grain, v./°F.
K_I = integral gain, v./(°F.)(sec.)
K_u = ultimate gain, v./°F.
L = process length ($L = 14.25$ ft.), ft.
P = dimensionless coefficient $\left(P = \dfrac{\overline{U}A_H}{pC_pA_c\overline{V}} = 0.85\right)$
P_c = pressure signal to pneumatic valve, lb./sq.in. gauge
P_u = ultimate period, seconds
r = dimensionless flow rate deviation from steady state $\left(r = \dfrac{v - \overline{v}}{v}\right)$
s = Laplace transform operator
t = dimensionless time $\left(t = \dfrac{\overline{v}t'}{L}\right)$
t' = real time, sec.
T_i = inlet temperature, °F.
T_{i0} = steady state inlet temperature, °F.
T_0 = outlet temperature, °F
T_r = reset rate, sec.
T_s = steam temperature, °F.
T_{sp} = set point temperature, °F.
$\overline{T}$ = time averaged temperature, °F.
U = overall heat transfer coefficient, B.t.u./(lb.)(sq.ft.)(°F.)
$\overline{U}$ = overall heat transfer coefficient at average flow rate ($\overline{U} = 688$), B.t.u./(lb.)(sq.ft.)(°F.)
v = water velocity, ft./sec.
$\overline{v}$ = average water velocity ($\overline{v} = 3.55$ ft./sec.), ft./sec.
x = dimensionless distance ($x = x'/L$)
x' = length, ft.
X = input
Y = output

Greek Letters

Δ = incremental distance
ϵ = deviation from set point temperature, °F.
ζ = dummy variable of integration
θ = manipulative variable dead time, sec.
ρ = water density, lb./cu.ft.
τ = dead time of process at steady state ($\tau = 4$ sec.), sec.
ϕ = temperature related dimensionless process dependent variable $= -\dfrac{1}{P}\ln\dfrac{T_s - T}{T_s - T_{10}} - x$

Subscripts

m = final output
n = counting index, reaction order, exponent

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